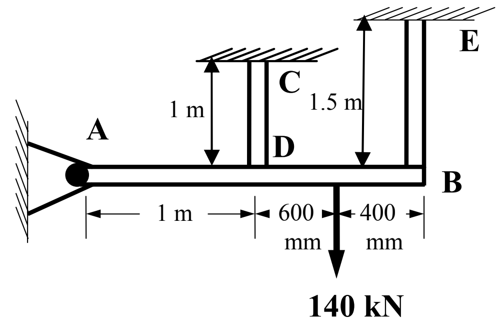
2006 GE 213.3 Midterm Exam Solutions

1. The bar *AB* is considered to be absolutely rigid and is horizontal before the load of 140 kN is applied, (Fig. 1). The connection at *A* is a pin, and *AB* is supported by the steel rod *EB* (E=200 GPa) and the copper rod *CD (E=120 GPa)*. The length of *CD* is 1 m, of *EB* is 1.5 m. The cross-sectional area of *CD* is 500 mm^2, and of *EB* is 300 mm^2. Determine the stress in each of the vertical rods and the deflection at the load point of bar *AB*. Neglect the weight of *AB*. *[δ =PL /AE] (14 Marks)*



Solution:

(a) Given:

Rod CD: Copper E= 120 GPa; Length = 1m and Area = 500 mm^2

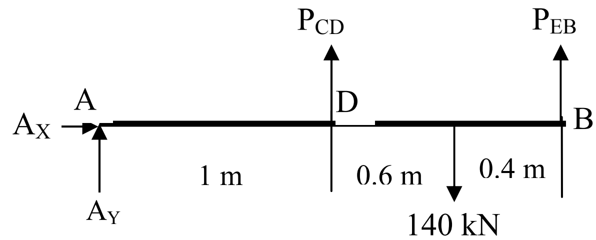
Rod EB: Steel E = 200 GPa; Length = 1.5 m and Area = 300 mm^2

Now the Free-Body Diagram of ADB and taking Moment about point A

$+\circlearrowleft \Sigma M_A = 0$

$1.0\ P_{CD} + 2.0\ P_{EB} - 1.6\ (140\ kN) = 0$

or $P_{CD} + 2\ P_{EB} = 224\ kN$ ------------*(1)*

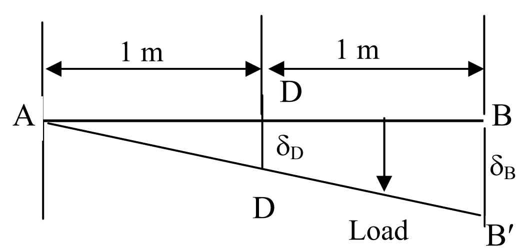


For another relation, we draw the deformation diagram (inderminate problem)

From similar triangles ADD′ and ABB′

$$\frac{DD'}{AD} = \frac{BB'}{AB} \quad or \quad \frac{\delta_D}{1} = \frac{\delta_B}{2}$$

or $\qquad 2\delta_D = \delta_B$ ------------*(2)*



Now, deflection, $\delta = \dfrac{PL}{AE}$

Therefore,

$$\delta_D = \frac{P_{CD}\ L_{CD}}{A_{CD}\ E_{CD}} = \frac{P_{CD}(1m)}{(500\ mm^2 \times 10^{-6})(120 \times 10^9\ Pa)} = 16.67 \times 10^{-9}\ P_{CD}$$

Similarly, $\delta_B = \dfrac{P_{EB}\,L_{EB}}{A_{EB}\,E_{EB}} = \dfrac{P_{EB}(1.5\,m)}{(300\,mm^2 \times 10^{-6})(200 \times 10^9\,Pa)} = 25.0 \times 10^{-9}\,P_{EB}$

Substituting in Eq. (2): $2\delta_D = \delta_B$

$$\frac{2 \times P_{CD}(1m)}{(500\,mm^2 \times 10^{-6})(120 \times 10^9\,Pa)} = \frac{P_{EB}(1.5\,m)}{(300\,mm^2 \times 10^{-6})(200 \times 10^9\,Pa)}$$

or $\boldsymbol{2P_{CD} = 1.5\,P_{EB}}$ ***or*** $\boldsymbol{P_{CD} = 3/4\,P_{EB}}$

Substituting this in Eq. (1): $\boldsymbol{P_{CD} + 2\,P_{EB} = 224\,kN}$, we get

$3/4\,P_{EB} + 2\,P_{EB} = 224\,kN$ *or* $\boldsymbol{P_{EB}} = (4/11)\,224\,kN = \boldsymbol{81.45\,kN}$

and $\boldsymbol{P_{CD}} = 3/4\,P_{EB} = \boldsymbol{61.10\,kN}$

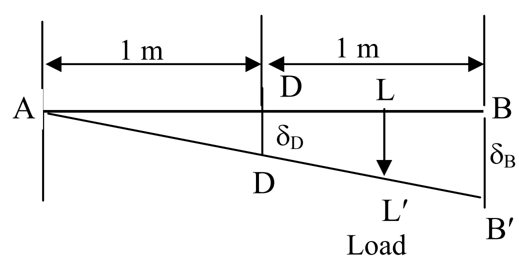
Now Stresses in rods

$$\sigma_{CD} = \frac{P_{CD}}{A_{CD}} = \frac{61.10\,kN}{500\,mm^2 \times 10^{-6}} = 122.18\,MPa \quad \Leftarrow \textbf{Ans.}$$

$$\sigma_{EB} = \frac{P_{EB}}{A_{EB}} = \frac{81.45\,kN}{300\,mm^2 \times 10^{-6}} = 271.5\,MPa \quad \Leftarrow \textbf{Ans.}$$

For Deflection of bar AB at the load point, using similar triangles

From similar triangles ALL′ and ABB′



$$\frac{LL'}{AL} = \frac{BB'}{AB} \quad or \quad \frac{\delta_L}{1.6} = \frac{\delta_B}{2}$$

Thus, $\delta_L = (0.8)\,\delta_B$

Substituting P_{EB} to determine δ_B

$$\delta_B = \frac{(81.45kN)(1.5\,m)}{(300\,mm^2 \times 10^{-6})(200 \times 10^9\,Pa)} = 2.036\,mm$$

Therefore, $\boldsymbol{\delta_L} = (0.8)\,\delta_B = \mathbf{1.63\ mm}\downarrow \quad \Leftarrow Ans.$

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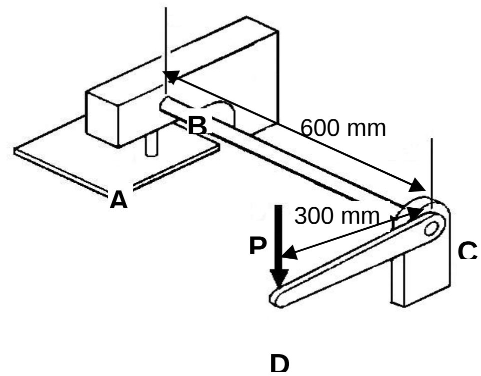
2. A hole is punched in a plastic sheet by applying 500-N force P to the end of lever CD, which is rigidly attached to the solid shaft BC. Design specification requires that the displacement of end D should not exceed 15 mm to complete the hole punching process. Determine the required diameter of shaft BC if the shaft is made of steel for which modulus if rigidity G = 75 GPa and allowable shear stress τ_{all} = 80 MPa. *[$\varphi = TL/JG$]. (12 marks)*

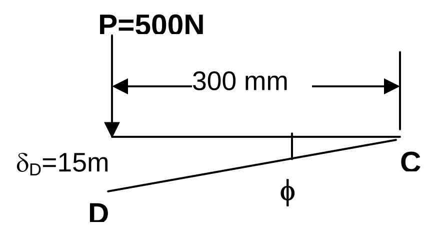


Solution

Data given:
For steel: G= 75 GPa; and Allowable shear stress, τ_{all} = 80 MPa
Length of the shaft BC = 600 mm = 0.6 m
δ_D = 15 mm at the end of 300 mm lever with a load of 500N (as shown in the sketch)

Therefore, Angle of twist on shaft BC, $\phi = \dfrac{15mm}{300mm} = 0.05\,rad$, and

Torque on the shaft BC, $T = (500N)(0.3m) = 150\ N.m$



$$J = \frac{\pi}{2}c^4$$

<u>Shaft diameter on the basis of angle of twist</u>

$$\phi = \frac{TL}{JG} = \frac{TL}{(\frac{\pi}{2}c^4)G} \ or\ J = \frac{TL}{G\phi} = \frac{(150\ N.m)(0.6m)}{(75\times10^9\ Pa)(0.05\ rad)} = 24.0\times10^{-9}\ m^4$$

$$\therefore c^4 = \frac{2TL}{\pi G\varphi} = \frac{2(180\ N.m)(0.6m)}{\pi(75\times10^9\ Pa)(0.05\ rad)} = 15.278875\ \times10^{-9}$$

Thus, c = 11.118 $\times10^{-3}$ m =11.118 mm or <u>d = 22.236 mm</u>

<u>Shaft diameter on the basis of allowable shear stress</u>

$$\tau = \frac{Tc}{J} = \frac{Tc}{(\frac{\pi}{2}c^4)} = \frac{2T}{\pi c^3} \ or\ \frac{J}{c} = \frac{T}{\tau} = \frac{150\ N.m}{80\times10^6\ Pa} = 1.875\ \times10^{-6}\ m^3$$

$$\therefore c^3 = \frac{2T}{\pi\tau_{all}} = \frac{2(150\ N.m)}{\pi(80\times10^6\ Pa)} = 1.1937\ \times10^{-6}$$

Thus, c = 10.608 $\times10^{-3}$ m =10.608 mm or <u>d = 21.216 mm</u>

We should select the larger value of diameter: $\therefore$ <u>d = 22.24 mm</u> $\Leftarrow$ *Ans*.

3. A steel rod of diameter 18 mm and length 4 m is held snugly (but without any initial stresses) between fixed walls by the arrangement shown in Fig. 3. Calculate the temperature drop T (degrees Celsius) at which the average shearing stress in the 16 mm diameter bolt becomes 50 MPa. (For steel, α = 12 x 10^{-6} / °C and E = 200 GPa). *[$\sigma_T = E\ \alpha\ \Delta T$] (12 marks)*

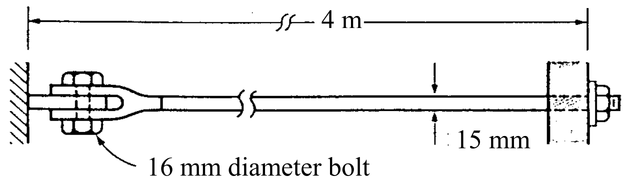


Figure 3

Solution:

Shear force in the bolt: d = 16 mm, in double shear

$$P_{bolt} = \tau \times 2A = (50 \times 10^6\ Pa) \times 2(\pi/4 \times 16^2 x 10^{-6}\ m^2)$$

$$= 6400\pi\ N = \mathbf{20{,}106.2\ N}$$

This force is caused by shrinking (drop in temperature) of the 4-m rod.
Therefore, ***compressive force*** in the rod due to drop in temperature

$$P_{rod} = \sigma_t.\ A = (E.\alpha.\ \Delta T)\ A = (200 \times 10^9\ Pa)(12 \times 10^{-6}\ /^0C)\ \Delta T\ (\pi/4 \times 18^2 x 10^{-6}\ m^2)$$

$$= 194.4\pi\Delta T = \mathbf{610.726\ \Delta T\ N}$$

However, the *compressive force* in the rod must be equal to the shearing force in the bolt, thus, -$P_{rod} = P_{bolt}$

$$-194.4\pi\ \Delta T = 6400\pi$$

$$\underline{\therefore\ \Delta T = -\ 32.92\ ^0C \Leftarrow ans.}$$

Alternante Solution:
The ***compressive force*** in the rod must be equal to the ***shearing force*** in the bolt,
Therefore: δ rod due to force = δ due to temperature decrease

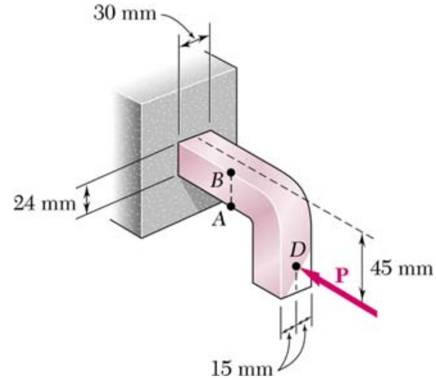
Or, $$\frac{PL}{AE} = \alpha.\Delta T.L \quad or\ .\Delta T = \frac{-P}{AE\alpha}$$

$$\therefore \quad \Delta T = \frac{-6400\pi\ N}{(\frac{\pi}{4}(18)^2 \times 10^{-6}\ m^2)(200 \times 10^9\ Pa)(12 \times 10^{-6}\ /^0\ C)} = -32.92^0\ C$$

<u>i.e, the temperature drop will be 32.92⁰ C. ⇐ ans.</u>

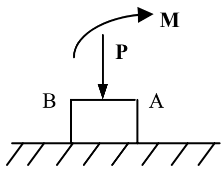
4. For the bracket shown in Fig. 4, the magnitude of the horizontal force **P** is 8 kN. Determine the stress at point (a) point A; (b) point B. *(12 Marks)*

Fig.4



Solution:

On section AB, this could be represented by the following diagram where A is in compression and B is in tension



For bending, AB(d) = 24 mm,
thus, c = 12 mm

Width BD (b) = 30 mm
Area= (30mm)(24mm) = 720 mm²=720×10⁻⁶ m²

$$Area = (30mm)(24mm) = 720\ mm^2 = 720\times10^{-6}\ m^2$$

e= 45mm – 12 mm= 33 mm
P = 8000 N

$$\therefore\quad M = (8000\ N \times 0.33\ m) = 264\ N.m$$

$$I = \frac{bd^3}{12} = \frac{(30mm)(24mm)^3}{12} = 34.56\times10^{-3}\,mm^4 = 34.56\times10^{-9}\,m^4$$

(a) Stress at Point A (compressive)

$$\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8\times10^3\,N}{720\times10^{-6}\,m^2} - \frac{(264N.m)(12\times10^{-3}\,m)}{34.56\times10^{-9}\,m^4}$$

$$= -11.11MPa \quad -91.67\,MPa \qquad = -\ 102.78\,MPa \qquad \underline{\Leftarrow \textbf{ans.}}$$

(b) Stress at Point B (tensile)

$$\sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8\times10^3\,N}{720\times10^{-6}\,m^2} + \frac{(264N.m)(12\times10^{-3}\,m)}{34.56\times10^{-9}\,m^4}$$

$$= -11.11\,MPa \quad +\ 91.67\,MPa \qquad = +\,80.56\,MPa \qquad \underline{\Leftarrow \textbf{ans.}}$$